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2002 J. Phys.: Condens. Matter 14 L249

(http://iopscience.iop.org/0953-8984/14/12/103)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS: CONDENSED MATTER

J. Phys.: Condens. Matter **14** (2002) L249–L256 PII: S0953-8984(02)33318-6

LETTER TO THE EDITOR

Current response in coupled chains under external electric and magnetic fields

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Received 1 February 2002
Published 15 March 2002
Online at stacks.iop.org/JPhysCM/14/L249

Abstract
Current response in coupling chains under a dc–ac electric field and a uniform magnetic field is investigated. For a special case of extremely low coupling between chains, simple and physically explicit analytical current expressions are derived, from which the field dependence of some physical properties can be clearly seen.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Ever since the pioneering work of Esaki and Tsu [1], which opened up a novel field of rich physics and potential device applications of semiconductor superlattices, these synthetic nanostructures have remained a focus of intensive research. A variety of very interesting phenomena, including Bloch oscillations [2], Wannier–Stark ladders [3], and Landau–Zener tunnelling [4], have been observed in high-quality superlattices [5, 6], optical ring resonators [7], and more recently for ultracold atoms in accelerating optical potentials [8], to mention but a few. With the addition of a strong ac field, a number of interesting effects have been predicted, such as dynamical localization [9], band collapse [10], and fractional Wannier–Stark ladders [11]. In recent years, there has been much intensive investigation of the effect of external electric and/or magnetic field(s) on various properties of charged particles in low-dimensional lattices [13], especially coupling chains and ladder compounds [12]. Zhang and Jing [14] proposed a ladder system (coupled chains; see figure 1) with a uniform magnetic field applied perpendicularly to it. Using a Green function method, they obtained the wavefunctions for both the case of perfect chains and that of imperfect chains, from which properties such as the phase boundary and current vortex structure of the localized condensation were obtained. Michael Creutz [15] has developed a ladder compound which consists of two coupled chains (a ladder), and obtained a lot of exciting results. Generally speaking, the two-dimensional lattices can only be studied numerically. However, for the case of coupled chains (a ladder) the

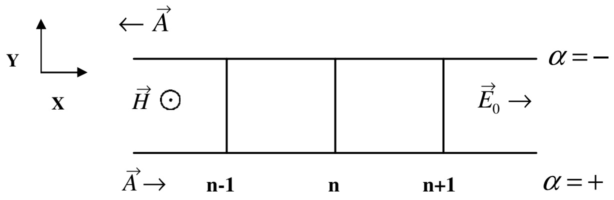

Figure 1. A tight-binding electron on coupled chains in a uniform electric field $\vec{E}$ and a uniform magnetic field $\vec{H}$. $\vec{A}$ is the vector potential.

problem can be solved analytically [14,15]. Nevertheless, up to now, no investigation of carrier transport properties of this model, with driving by a dc–ac electric field and a uniform magnetic field, with the help of Boltzmann's equation at finite temperatures has been performed. In this letter, we communicate our findings on this matter. For the special case of extremely low coupling between the chains, we find simple and physically explicit analytical expressions for the long-time average current of the system. Our results shows clearly that for the limit of extremely low coupling, a magnetic field can smear the Esaki–Tsu peak under some conditions. From the analytical expressions and numerical calculation of the long-time average current of this model, we find that when the ratio $edE/\hbar\omega_{ac}$ are the roots for the Bessel function J_0 (e is the electronic charge, d is the lattice constant, and J_0 is the Bessel function of order zero), the electron motion will have a tendency towards localization.

2. Hamiltonian

We start with a spinless electron on the basic structure in the presence of a perpendicular magnetic field as shown in figure 1. All horizontal bonds are of equal strength, given by the hopping parameter V. For vertical bonds, the coupling is V_0. Applying a perpendicular magnetic field induces phases when a particle hops around closed loops. Adopting the convention of placing these factors on the upper and lower horizontal bonds, we assume that these parameters are positive. Thus, the system Hamiltonian [15] is

$$H = -\sum_{n,\alpha}[V_0|n,\alpha\rangle\langle n,-\alpha| + V(\mathrm{e}^{\mathrm{i}\alpha\gamma/2}|n,\alpha\rangle\langle n+1,\alpha| + \mathrm{e}^{-\mathrm{i}\alpha\gamma/2}|n+1,\alpha\rangle\langle n,\alpha|)]. \quad (1)$$

The value of γ is determined by the magnetic field through the relation $\gamma = 2\pi\phi/\phi_0$, where ϕ_0 is the flux quantum (hc/e) and ϕ is the flux passing through a unit cell. A general one-particle state is (henceforth we set $\hbar = 1$ and the lattice constant $d = 1$)

$$|\Psi\rangle = \sum_{m,\alpha} C_{m,\alpha}|m,\alpha\rangle = \sum_{k} C_{k,\alpha}|k,\alpha\rangle \quad (2)$$

where $C_{k,\alpha} = \sum_m \mathrm{e}^{\mathrm{i}mk} C_{m,\alpha}$. For an infinite chain, this model is easily solved via Fourier transformation. Thus, the system's Hamiltonian in k-space is

$$H = -\int_0^{2\pi} \frac{\mathrm{d}k}{2\pi} \begin{pmatrix} 2V\cos(k-\gamma/2) & V_0 \\ V_0 & 2V\cos(k+\gamma/2) \end{pmatrix} |k,\alpha\rangle\langle k,\alpha| \quad (3)$$

and the energy eigenvalues are

$$\varepsilon_{\pm}(k) = -2V\cos(k)\cos(\gamma/2) \pm \sqrt{4V^2\sin^2(k)\sin^2(\gamma/2) + V_0^2}. \quad (4)$$

From the above equation, we can first calculate the macroscopic-average velocity of the Bloch electron, and determine the distribution $f(t)$ with the help of both the Boltzmann equation and the equilibrium distribution f_0; finally, we obtain the expressions for currents in the system.

For convenience in our investigation, we can divide the strength of coupling between chains into three classes: weak, intermediate, and strong coupling. For the first case we can deal with the weak coupling between chains as a small perturbation for the upper and low chains respectively. For the third case the strong-coupling chains can be reasonably taken as a new single chain in which every new site is a pair consisting of an upper site and a corresponding low site. With the help of well-known results on the single chain, we can fully investigate these two cases. For the second case it is difficult to find a suitable analytical method. To set a reference frame for later discussions, let us limit our investigation to the first case. There are two reasons for doing this: one is that this case can be considered analytically; the other (very important) is that this case is close to the situation for real coupling chains commonly encountered. Let us first look at the transient currents of the system.

2.1. Transient currents

The macroscopic-average state may be specified by a distribution function $f^s(t)$ that should satisfy the Boltzmann equation

$$\frac{\partial f^s(k,t)}{\partial t} + eE(t)\frac{\partial f^s(k,t)}{\partial k} = St(f^s(k,t)), \tag{5}$$

where $E(t)$ is the dc–ac electric field, and $s = \pm$ correspond to $\varepsilon_\pm(k)$ respectively. $St(f^s(k,t))$ denotes the collision integral. For the simplicity, the collision integral can be put into the form [16]

$$St(f^s(k,t)) = -\nu_{ie}(f^s(k,t) - f_0^s) - \nu_e(f^s(k,t) - f^s(-k,t)), \tag{6}$$

where f_0 is the equilibrium distribution function, and ν_{ie} and ν_e are the relaxation frequency of the energy and the frequency of elastic collisions respectively. Due to the periodicity of the energy spectrum, we can transform the Boltzmann equation (BE) in k-space into the BE expression in real space by using $f^s(k,t) = \sum_n f_n^s(t)\mathrm{e}^{\mathrm{i}nk}$, where $f_n^s(t) = \int_0^{2\pi}(\mathrm{d}k/2\pi)\, f^s(k,t)\mathrm{e}^{-\mathrm{i}nk}$. Thus, the Fourier transforms $f_n^s(t)$ satisfy the following equations:

$$\frac{\mathrm{d}f_n^s(t)}{\mathrm{d}t} + \mathrm{i}neE(t)f_n^s(t) = -\nu_{ie}(f_n^s(t) - f_{0,n}^s), \tag{7}$$

where

$$f_{0,n}^s = \frac{\int_0^{2\pi}\,\mathrm{d}k\exp(-\beta\varepsilon_s(k))\cos(nk)}{\int_0^{2\pi}\,\mathrm{d}k\exp(-\beta\varepsilon_+(k)) + \int_0^{2\pi}\,\mathrm{d}k\exp(-\beta\varepsilon_-(k))} \tag{8}$$

are Fourier transforms of the Boltzmann equilibrium distribution function f_0^s.

For the sake of simplicity, we first focus on the case of $\nu_e = 0$ in this letter. Clearly, equation (7) can be solved exactly. If a dc–ac field $E(t) = E_0 + E_1\cos(\omega t)$ is applied to the coupling chains at $t = 0$, the solutions of equation (7) read

$$f_n^s(t) = \exp(-\nu_{ie}t - \mathrm{i}nA(t))\int_0^t \mathrm{d}t'\nu_{ie}f_{0,n}^s\exp(\nu_{ie}t' + \mathrm{i}nA(t')), \tag{9}$$

where

$$A(t) = e\int_0^t \mathrm{d}t'\,E(t') = \omega_B t + \frac{eE_1}{\omega}\sin(\omega t), \qquad \omega_B = eE_0. \tag{10}$$

From equation (4) we can calculate the electron's quasi-classical velocity from $v(k) = \partial\varepsilon(k)/\partial k$. Hence, the current density is written as

$$j^s(t) = \int_0^{2\pi} ev(k)f^s(k,t)\,\mathrm{d}k, \tag{11}$$

and then the total transient current is

$$j(t) = \sum_s j^s(t) = \sum_s e\int_0^{2\pi} v(k) f^s(k,t)\,\mathrm{d}k = j_a^s(t) + j_b^s(t) \tag{12}$$

where

$$j_a^s(t) = \mathrm{i}\,2\pi eV\cos(\gamma/2)(f_1^s(t) - f_{-1}^s(t)) = -4\pi eV\cos(\gamma/2)\,\mathrm{Im}(f_1^s(t)), \tag{13}$$

$$j_b^s(t) = se\int_0^{2\pi}\left(\frac{2V^2\sin^2(\gamma/2)\sin(2k)}{\sqrt{4V^2\sin^2(\gamma/2)\sin^2(k) + V_0^2}}\right) f^s(k,t)\,\mathrm{d}k. \tag{14}$$

The general case of the system current $j_b^s(t)$ can hardly be considered analytically, because this current $j_b^s(t)$ has a very complex expression. Fortunately, we can investigate analytically the special case of extremely weak coupling ($V_0 \ll V$), which is commonly encountered in real coupling chains. In the weak-coupling approximation, i.e., $V_0 \ll V$, the current $j_b^s(t)$ has a very simple expression. In this case, the transient current $j_b^s(t)$ and $j^s(t) = j_a^s(t) + j_b^s(t)$ can be rewritten as

$$j_b^s(t) = e\int_0^{2\pi}\left(\frac{\partial}{\partial k}s(2V\sin(k)\sin(\gamma/2))\right) f^s(k,t)\,\mathrm{d}k = s\,4\pi eV\sin(\gamma/2)\,\mathrm{Re}(f_1^s(t)), \tag{15}$$

and

$$j_a^s(t) = -4\pi eV\cos(\gamma/2)\,\mathrm{Im}(f_1^s(t)). \tag{16}$$

The transient current $j^s(t)$ of the system reads

$$j^s(t) = j_a^s(t) + j_b^s(t) = -4\pi eV\cos(\gamma/2)\,\mathrm{Im}(f_1^s(t)) + s\,4\pi eV\sin(\gamma/2)\,\mathrm{Re}(f_1^s(t)), \tag{17}$$

and the total transient current $j(t)$ of the system is

$$j(t) = \sum_s j^s(t).$$

From the experimental viewpoint, the physical current observable in experiment is the long-time average current instead of the transient current. Therefore, we focus our attention on the long-time average current of the system in the following discussion.

2.2. *Long-time average currents ($V_0 \ll V$)*

Once the transient current $j(t)$ has been obtained, the long-time average current of the system can be calculated easily by using the relation $j = \langle j(t)\rangle = \lim_{T\to\infty}(1/T)\int_0^T \mathrm{d}t\, j(t)$. Using the relation of $f_{-n}^s = (f_n^s)^*$ which can be obtained from (9), we can simplify the current $j^s(t)$ expression to the following form:

$$j^s(t) = -\cos(\gamma/2)\,\mathrm{Im}(f_1^s) + s\sin(\gamma/2)\,\mathrm{Re}(f_1^s), \tag{18}$$

$$j^s = \langle j^s(t)\rangle = 4\pi eV\sum_l f_{01}^s J_l^2(eE_1/\omega)\frac{\omega\tau(\omega_B/\omega - l)\cos(\gamma/2) + s\sin(\gamma/2)}{(\omega\tau)^2(\omega_B/\omega - l)^2 + 1}, \tag{19}$$

where J_l is the ordinary Bessel function and $\tau = 1/v_{ie}$. Finally, the long-time average current j is

$$\begin{aligned} j/j_0 &= \sum_s\sum_{l=-\infty}^{+\infty} f_{0,1}^s J_l^2(eE_1/\omega)\frac{\omega\tau(\omega_B/\omega - l)\cos(\gamma/2) + s\sin(\gamma/2)}{(\omega\tau)^2(\omega_B/\omega - l)^2 + 1} \\ &= \sum_{l=-\infty}^{+\infty} J_l^2\frac{(f_{0,1}^+ + f_{0,1}^-)\omega\tau(\omega_B/\omega - l)\cos(\gamma/2) + (f_{0,1}^+ - f_{0,1}^-)\sin(\gamma/2)}{(\omega\tau)^2(\omega_B/\omega - l)^2 + 1}, \end{aligned} \tag{20}$$

where $j_0 = 4\pi eV$.

Equations (4), (8), (9), and (20) are the central results of this letter; in these, the field dependence of some physical properties can be clearly seen. In the next section we elucidate the relation of the long-time average currents to the system's parameters.

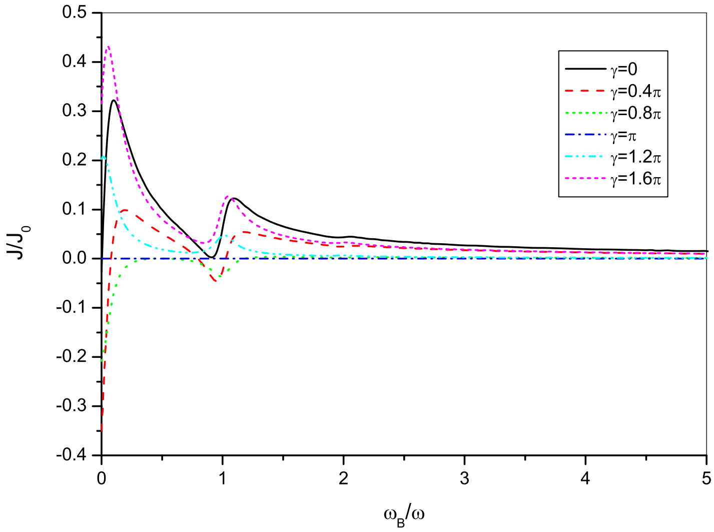

Figure 2. The magnetic effect on the long-time average current j/j_0 for the case of weak coupling between chains ($V/\omega = 1.0$, $V_0/\omega = 0.1$). The parameters are $eE_1/\omega = 1.0$, $\upsilon_{ie}/\omega = 0.1$, $\beta\omega = 1.5$, and $\gamma = 0.0$, $2\pi/5$, $4\pi/5$, and $8\pi/5$.

3. Results and discussion

At the outset, we would like to note that the system current consists of an upper current and a lower current corresponding to the upper energy state (ε_+) and lower energy state (ε_-) respectively. The two currents are not independent due to the system's distribution (8). Thus, we can change the strength of the coupling between the chains to control the current of the coupled-chain system. We would also like to note that we have to set our temperature parameter as room temperature, because we employed the Boltzmann distribution in our discussion. If we chose low temperature, for example $T < 20$ K, we would have to employ the Fermi–Dirac distribution in the discussion. Now, let us consider the case of the effect of an abruptly applied field $E(t) = E_0 + E_1\cos(\omega t)$ on the coupling chains ($V_0 \ll V$) at $t = 0$ under the above assumption.

To illustrate the phenomena fully in this case, and as a first step, we shall investigate the magnetic effect on the long-time average current in a system of weakly coupling chains. Figure 2 displays an example for $eE_1/\omega = 0.8$, where the coupling parameters are $V_0/\omega = 0.1$, $V/\omega = 1.0$, the inelastic scattering frequency is $\upsilon_{ie}/\omega = 0.1$, the temperature parameter is $\beta\omega = 1.0$, and the magnetic parameters are $\gamma = 0.0$, $2\pi/5$, $4\pi/5$, π, $6\pi/5$, and $8\pi/5$. From the figures, we can see that the Esaki–Tsu and integer peaks appear, just as for the single chain, near the positions of $\omega_B/\omega = 1$ and 2. However, the fractional peaks which appear for the single chain disappear. This phenomenon can be understood from observing the curve in the figure for $\gamma = 0.0$: we can see that it is unaffected by the magnetic field. Another remarkable thing is that if we increase the value of γ from 0 to π, the Esaki–Tsu peak and integer peaks become lower and lower, and actually disappear if $\gamma = \pi$; and if we go on increasing the value of γ from π to 2π, the Esaki–Tsu peak and integral peaks emerge again, but in the negative direction. In short, a magnetic field can totally smear both the Esaki–Tsu peak and the integer peaks. Magnetic field is not one of the factors that smear the fractional peaks.

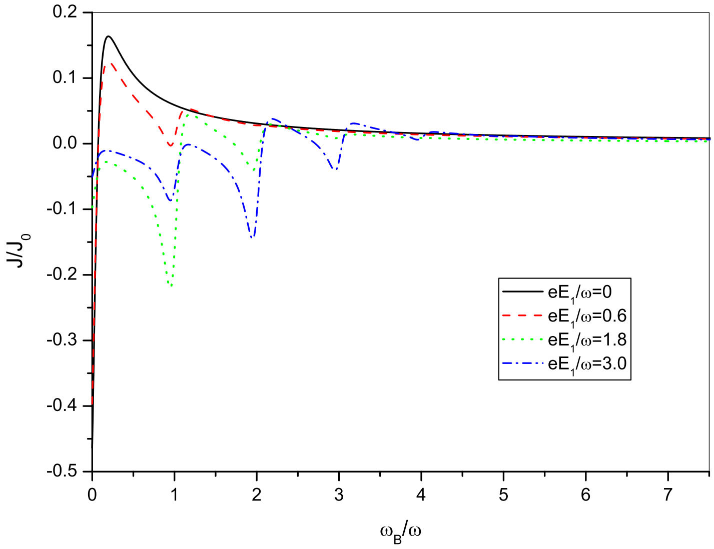

Figure 3. The magnetic effect on the long-time average current j/j_0 for the case of weak coupling between chains ($V/\omega = 1.0$, $V_0/\omega = 0.1$). The parameters are $eE_1/\omega = 1.0$, $\upsilon_{ie}/\omega = 0.1$, $\beta\omega = 1.5$, and $\gamma = 0.0$, $2\pi/5$, $4\pi/5$, and $8\pi/5$.

Second, we observe the ac electric field effect on the long-time average current in the weak-coupling approximation. Figure 3 shows the case for magnetic parameter $\gamma = 2\pi/5$, where the other parameters are $\upsilon_{ie}/\omega = 0.1$, $V/\omega = 1.0$, $V_0/\omega = 0.1$, and $\beta\omega = 1.0$. We can note that with increasing eE_1/ω the Esaki–Tsu peak becomes lower and lower; in the meantime, the integral peaks emerge gradually. From the figures, we also note that the bigger the value of eE_1/ω, the stronger the effect of a dc–ac electric field on the Esaki–Tsu peak. From the findings for $eE_1/\omega = 0.0$, one can note that a dc–ac electric field is also not a factor that smears the fractional peaks. But if we choose $eE_1/\omega = 2.405$ or 5.5201, the Esaki–Tsu peaks almost disappear and small integer peaks emerge, as illustrated in figure 3. If $eE_1/\omega = 3.8317$, the electric fields can smear the first integer peak, as shown in figure 4. These findings are just a manifestation of a tendency towards localization [9].

Finally, figures 2–4 show that the fractional peaks, which appear for the single chain, disappear due to the coupling between chains rather than because of the application of a uniform magnetic field or a dc–ac electric field. Furthermore, the most remarkable thing is that the total current of the coupling chains is negative when $\omega_B/\omega = 0.0$ (i.e., in the absence of a static electric field), and the magnitude of the current is dependent on γ. The reason for this, we think, is that a magnetic field can vary the system's energy spectrum, which determines the distribution of electrons in the coupling chains, and thus change the magnitudes of the upper and lower currents corresponding to the upper energy (ε_+) and lower energy (ε_-) of the coupling chains, consequently leading to a negative value of the total current of the system.

4. Conclusions

In conclusion, within a simple model of a tight-binding Bloch electron on coupled chains under the influence of both a uniform magnetic field and a dc–ac electric field, we have studied the

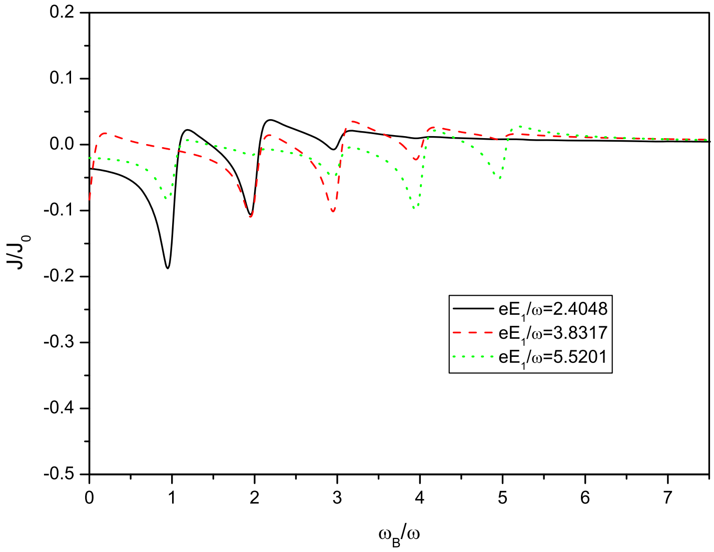

Figure 4. The magnetic effect on the long-time average current j/j_0 for the case of weak coupling between chains ($V/\omega = 1.0$, $V_0/\omega = 0.1$). The parameters are $eE_1/\omega = 1.0$, $\upsilon_{ie}/\omega = 0.1$, $\beta\omega = 1.5$, and $\gamma = 0.0$, $2\pi/5$, $4\pi/5$, and $8\pi/5$.

electric current response of the system. We have shown the variation in height of these peaks, as well as in the total-current magnitude with temperature and with the various parameters of the theory, including the magnetic and dc–ac electric fields. Our results show that we can control the Esaki–Tsu peak by changing the magnetic flux per cell. This can be realized in practice using a dimerized semiconductor superlattice by, for example, inserting a 'coupling' semiconductor layer into the two semiconductor superlattices. Adjusting the thickness of the 'coupling' layer, one could test the predictions of this letter for this system by varying the external uniform magnetic and dc–ac electric field.

We would like to thank Dr Wei Zhang, Dr Suqing Duan and Dr Ying Luo for many fruitful discussions. This work was supported in part by the National Natural Science Foundation of China under grants No 19725417 and No 10174095, the National PAN-DENG Project under grant No 95-YU-41, and a grant from the China Academy of Engineering Physics.

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